PH16212, Homework 2

Deadline: Oct. 14, 2019

(Conventions) Five-point momentum twistor parametrization:

$$
\begin{gathered}
\langle 12\rangle \to 1, \langle 13\rangle \to 1, \langle 14\rangle \to 1, \langle 15\rangle \to 1, \langle 23\rangle \to -\frac{1}{x_1}, \langle 24\rangle \to -\frac{1}{x_2}-\frac{1}{x_1} \\
\langle 25\rangle \to -\frac{1}{x_2}-\frac{1}{x_3}-\frac{1}{x_1}, \langle 34\rangle \to -\frac{1}{x_2}, \langle 35\rangle \to -\frac{1}{x_3}-\frac{1}{x_2}, \langle 45\rangle \to -\frac{1}{x_3} \\
[12] \to -x_1, [13] \to x_1+x_2x_5, [14] \to -x_5x_3+x_3-x_2x_5, [15] \to x_3\left(x_5-1\right), [23] \to x_1x_2x_4, \\
[24] \to -x_1\left(x_4x_3-x_3+x_2x_4\right), [25] \to x_1x_3\left(x_4-1\right) \\
[34] \to -x_1x_3+x_1x_4x_3+x_2x_4x_3-x_2x_5x_3+x_1x_2x_4, \\
[35] \to -x_3\left(x_4x_1-x_1+x_2x_4-x_2x_5\right), [45] \to x_2x_3\left(x_4-x_5\right)
\end{gathered} \tag{1}
$$

1. Explicitly prove the BCJ relation for the four-point gluon tree amplitude, for arbitrary helicity configurations.

$$sA(1342) = tA(1324)\,. \tag{2}$$

(Hint: all non-vanishing four-point gluon tree amplitudes are MHV amplitudes)

2. Use BCFW recursion relation to calculate the MHV and $\overline{\text{MHV}}$ amplitude,

$$A(1^-2^-3^+4^+5^+), \quad A(1^+2^-3^+4^-5^-). \tag{3}$$

3. Simplify

$$\frac{s_{12}A(2^-1^-3^+4^+5^+) - s_{23}A(1^-3^+2^-4^+5^+)}{A(1^-3^+4^+2^-5^+)} \tag{4}$$

to a functions in s_{12}, s_{23}, s_{34}, s_{45} and s_{15}. (Hint: You may use momentum twistor to calculate this ratio. The result will be a rational function of $x_1, \ldots x_5$. Simplify it with a computer algebra software. Then you can either directly guess the form in terms of s_{12}, s_{23}, s_{34}, s_{45} and s_{15}, or use my package's command ReduceFraction to get the conversion.)

4. Explicitly prove,

$$s_{12}s_{34}A(1^-2^-3^+4^+5^+)A(2^-1^-4^+3^-5^+) + s_{13}s_{24}A(1^-3^+2^-4^+5^+)A(3^-1^-4^+2^-5^+) = 0\,. \tag{5}$$